Dale Mellor
Scientific Software Developer
Cosmology Group
University of Manchester

Bespoke C++/Python implementation of cosmological theory

Previously:

Hydrology (finite element methods) and Hydrometeorology (stochastic modelling) parameters are known), may be used to solve for the unknown parameters $\bar{\nu}$, ρ and M, the average rate of occurrence of raincell births in unit time in unit area, the average rate of occurrence of the cluster functions on the inclined bands, and the maximum intensity of the rainfall inside a raincell, respectively.

Thus the expressions for the internal covariance structure are first rearranged into a convenient form, which shows explicitly the dependence of the covariance structure on the three model parameters in question, and then it is shown how the resulting system can be manipulated to give the required estimates of the model parameters. This is followed by an investigation into the number of intervals required by Simpson's rule to accurately evaluate the integrals involved in the numerical evaluation of the theoretical expressions. Having done this satisfactorily, a set of Monte-Carlo realizations of the MTB Model are generated using selected values for the model parameters. Then the estimation of $\bar{\nu}$, ρ and M is attempted using the internal covariance structure, assuming all the other parameters are known. Comparing the three estimated parameters with their true values then indicates the accuracy of the estimation procedure. Finally, the theory is applied to the one real storm which has been isolated from the Ingham radar data.

The internal covariance structure of the MTB Model may be written as

$$\gamma_{\underset{\sim}{k}}[\zeta] = \iint_{-\infty}^{\infty} \left(\gamma'_{\underset{\sim}{k}-\underset{\sim}{y}+\underset{\sim}{z}}[\nu] \, \frac{A^2}{4\pi} - \bar{\nu}^2 \right) g'(\underset{\sim}{y}) \, g'(\underset{\sim}{z}) \left(\frac{4M}{d^2 w^2} \right)^2 d^3\underset{\sim}{y} \, d^3\underset{\sim}{z} \quad (3\text{-}14)$$

$$+ \, \bar{\nu} \int_{-\infty}^{\infty} g'(\underset{\sim}{z}) \, g'(\underset{\sim}{z}+\underset{\sim}{k}) \left(\frac{4M}{d^2 w^2} \right)^2 d^3\underset{\sim}{z}$$

where

$$\gamma'_{\underset{\sim}{k}}[\nu] = \left(4\rho^2 + \rho Z_2(\underset{\sim}{k})\right) Z_1(\underset{\sim}{k}) \quad (3\text{-}15)$$

$$Z_1(\underset{\sim}{k}=(k_x,k_y,k_t)) = \frac{I(k_\gamma<\pi)}{\pi-k_\gamma} \left(\pi \cos k_\delta + 2\pi\right) \left[\left(\pi-k_\gamma\right) \cos k_\gamma + \sin^2 k_\gamma \left(\sin k_\gamma - \cos k_\gamma\right)\right] \quad (3\text{-}16)$$

$$Z_2(\underset{\sim}{k}) = \frac{3}{160 \, l^6} \left[I(k_\alpha<2\,l) \, \Gamma_\alpha + I(k_\beta<2\,l) \, \Gamma_\beta \right] \quad (3\text{-}17)$$

$$\Gamma_\Xi = \left(k_\Xi^2 + 3k_\Xi l + l^2\right) \left(l - k_\Xi\right)^3 \quad , \quad \Xi = \alpha \, , \, \beta \quad (3\text{-}18)$$

$$k_\alpha = k_x \cos\alpha + k_y \sin\alpha - c_a k_t \quad (3\text{-}19)$$

$$k_\beta = k_x \cos\beta + k_y \sin\beta - c_b k_t \quad (3\text{-}20)$$

$$k_\gamma = \frac{\pi}{L}\left(k_x - c_s k_t\right) \quad (3\text{-}21)$$

$$k_\delta = \frac{2\pi}{W}\left(k_x - c_B k_t\right) \quad (3\text{-}22)$$

$$A = \frac{\pi\bar{\nu}}{4\rho} \quad (3\text{-}23)$$

$$g'(x,y,t) = h'\left(x-c_x t \, , \, y-c_y t \, , \, t\right) \quad (3\text{-}24)$$

$$h'(x,y,t) = t\left(w^2 - 4x^2 - 4y^2\right)(d-t) \quad , \quad \sqrt{x^2+y^2} < \frac{w}{2} \quad , \quad 0 < t < d \quad (3\text{-}25)$$

(already used in eqn (2.36) on p. 38)

and where I is the indicator function taking the value unity if its argument is true, zero when false, and a list of all the model parameters is to be found in Table 2-1.

This can be further rearranged to give

$$\gamma_{\underset{\sim}{k}}[\zeta] = Y_1(\underset{\sim}{k}) \, M^2\bar{\nu}^2 + Y_2(\underset{\sim}{k}) \, \frac{M^2\bar{\nu}^2}{\rho} + Y_3 \, M^2\bar{\nu}^2 + Y_4(\underset{\sim}{k}) \, M^2\bar{\nu} \quad (3\text{-}26)$$

where

$$Y_1(\underset{\sim}{k}) = \frac{\pi}{d^4 w^4} \iint_{-\infty}^{\infty} Z_1(\underset{\sim}{k}-\underset{\sim}{y}+\underset{\sim}{z}) \, g'(\underset{\sim}{y}) \, g'(\underset{\sim}{z}) \, d^3\underset{\sim}{y} \, d^3\underset{\sim}{z} \quad (3\text{-}27)$$

(not: $Z_1(\underset{\sim}{k}-\underset{\sim}{y}+\underset{\sim}{z}) Z_2(\underset{\sim}{k}-\underset{\sim}{y}+\underset{\sim}{z})$?)

$$Y_2(\underset{\sim}{k}) = \frac{\pi}{4d^4 w^4} \iint_{-\infty}^{\infty} Z_1 Z_2(\underset{\sim}{k}-\underset{\sim}{y}+\underset{\sim}{z}) \, g'(\underset{\sim}{y}) \, g'(\underset{\sim}{z}) \, d^3\underset{\sim}{y} \, d^3\underset{\sim}{z} \quad (3\text{-}28)$$

$$Y_3 = -\frac{16}{d^4 w^4} \iint_{-\infty}^{\infty} g'(\underset{\sim}{y}) \, g'(\underset{\sim}{z}) \, d^3\underset{\sim}{y} \, d^3\underset{\sim}{z} \quad (3\text{-}29)$$

$$Y_4(\underset{\sim}{k}) = \frac{16}{d^4 w^4} \int_{-\infty}^{\infty} g'(\underset{\sim}{z}) \, g'(\underset{\sim}{z}+\underset{\sim}{k}) \, d^3\underset{\sim}{z} \quad (3\text{-}30)$$

It can be shown that

$$Y_4(k_x,0,0) = \frac{2d}{135\,w^4}\left[3w^4\left(6k_x^2-w^2\right)\left(2\sin^{-1}\frac{k_x}{w} - \pi\right) - 2k_x\sqrt{w^2-k_x^2}\left(4k_x^4-16k_x^2w^2-3w^4\right)\right] \quad \text{for } k_x<w \quad (3\text{-}31)$$

Note that this gives the required coefficient Y_4 for the evaluation of the covariance with spatial displacement k_x in the direction of the storm movement (recall that the theory of the internal covariance of the MTB Model, Section 2-3-3, assumes the storm travels from west to east), but, with no temporal lag involved, the covariance structure will be approximately isotropic at small displacements k_x, as the covariance structure will depend mainly on the raincells, which are circular. In fact, the value of k used here is 2 kilometres, the resolution of the radar data, which is smaller than the width of the raincells as required (Equation (3-31) is invalid when $k_x>w$ since the inverse sine is then undefined). Thus, in measuring this lagged covariance, the lag is always taken in the x-direction of the data, regardless of the orientation of the storm.

It can also be shown that

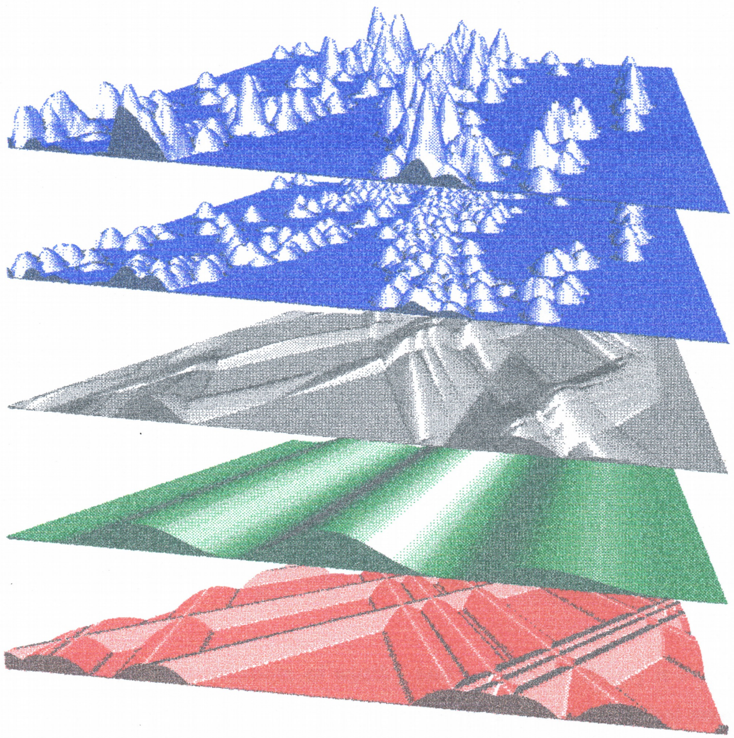

Figure 2-3 A typical realization of the model.

The University of Manchester

In outputting results of theory-implementing code:

Crucial to display in as much insightful detail as possible the inner workings of the code